# Resistance components, scaling and dimensional analysis

## Objectives

After working through these notes you should be able to:

1. identify and describe the main components of resistance
2. apply the Buckingham $\Pi$ theorem to dimensional analysis of ship resistance
3. outline the ship scaling problem

## References:

Lewis E. (Ed.) (1988) *Principles of Naval Architecture* Vol 2. Society of Naval Architects and Marine Engineers, New York

Benford H. (1991) *Naval Architecture for Non-Naval Architects*. Society of Naval Architects and Marine Engineers, New York.

Rawson, K. J. and Tupper, E. C. (2001) *Basic ship theory,* Butterworth-Heinmann, Oxford.

## 1 Introduction:

The resistance of a vessel to forward motion is one of the most fundamental hydrodynamic properties. A Naval Architect must be able to estimate the resistance at a reasonably early stage in the design process and constantly improve their estimate as the design progresses.

The accurate prediction of resistance has a number of advantages including early knowledge of the necessary engine size in terms of space requirement, weight, initial cost and fuel consumption. Therefore it is possible to design the optimum propulsion system eg. propeller.

An improvement in vessel resistance, which leads to minimum resistance at the required design speed, is also beneficial since it can reduce capital cost as well as running costs. However this requirement conflicts with many other requirements of ship design and, as is usual in the design process, results in a compromise.

Therefore, it is important to understand the causes of ship resistance, both to enable a good prediction to be made, and to design an efficient hull form within the many constraints and requirements imposed.

It is important to realise that all the components of ship resistance are not yet fully understood, the major complication being that they operate on the interface between water and air. Research is continuing year by year which improves our knowledge and leads to more efficient hull forms and better prediction techniques.

## 2 The Components of Resistance:

The total resistance of a vessel is made up of many individual factors.

The two principal components of resistance are the skin friction resistance and the wave making resistance.

The relationship between all the components is shown below:

*Fill in from class discussion*

**Skin friction resistance:** This is the component of resistance obtained by integrating the tangential forces over the hull surface. In other words, it is simply the effect of friction between the water and the ship's hull.

**Wavemaking resistance:** When a vessel moves along the surface of the water it creates a pattern of waves. Energy is required to create these waves and the wavemaking resistance is the component of resistance associated with the expenditure of this energy.

**Viscous pressure resistance:** This is the component of resistance obtained by integrating pressures due to the thickness of the boundary layer and wake. It is essentially a drag due to viscosity other than skin friction (i.e. form drag).

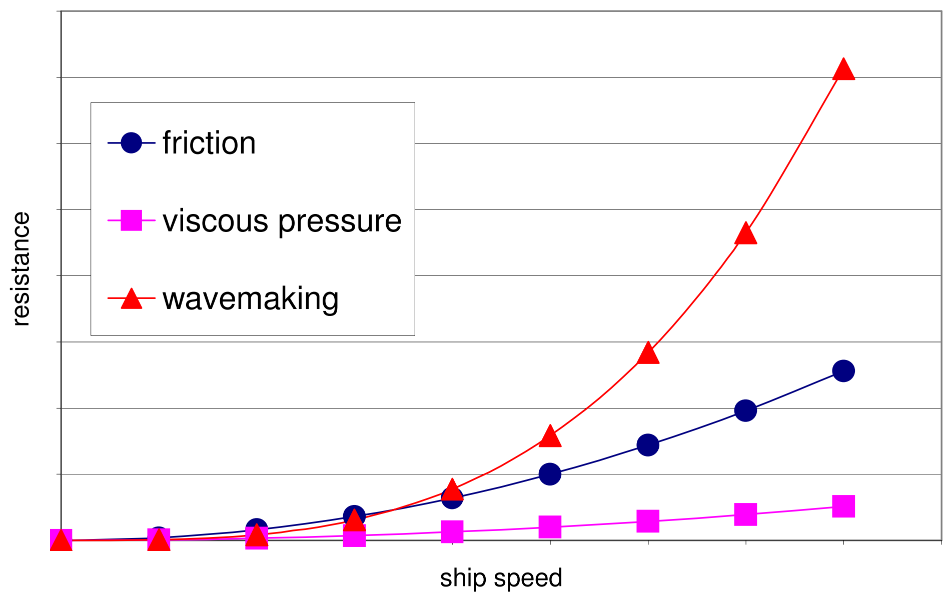
**Wave breaking resistance:** This is the component which arises from the energy expended in generating turbulence from the breaking of waves in the fore body near the bow. (It is sometimes included in the wavemaking resistance).

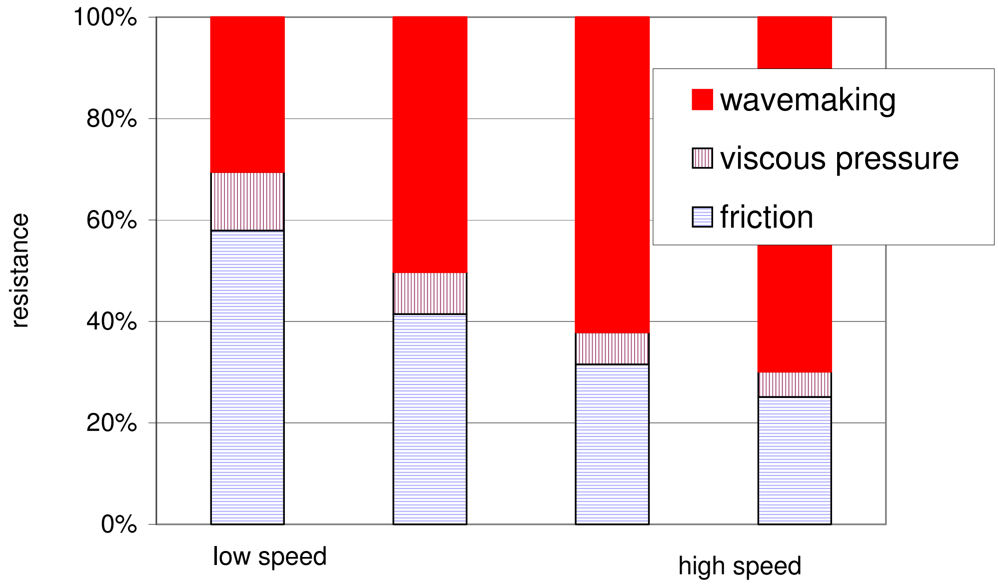
**Air resistance:** Resistance of the above water parts of the vessel due to the air flow. This resistance will comprise both frictional and eddy-making components.

**Appendage resistance:** All the above components are related only to the "naked" hull. The resistance of appendages (prop shafts & brackets, rudders, bilge keels etc) are considered separately.

## 3 Effect of speed on resistance:

At low speeds, the waves made by a vessel are very small and the resistance is almost wholly viscous in nature. As speed is increased the viscous resistance increases with speed to some power less than 2. However, the wavemaking resistance increases greatly with speed with a very steep increase occurring when the ship is travelling at the speed of a wave of its own length. In this case there will be a large crest at the bow and the stern with a deep trough amidships. This speed is sometimes known as maximum hull speed because of the power required to exceed it.






Thus, the resistance of vessels travelling slowly is predominantly viscous; skin friction and viscous pressure, whereas the resistance of a high speed displacement craft is predominantly wavemaking. Skin friction is strongly related to wetted surface area and hence the naval architect will concentrate on reducing this when designing a slow vessel. Since a sphere has the smallest surface area for a given volume, slow moving vessels will tend to be short and beamy, eg. tugs.
However a high speed displacement vessel will be designed as long as possible bearing in mind any possible interference effects.

## 4 Froude's Law of Comparison:

The scientific treatment of resistance was initiated by William Froude in the 1860's. His first and basic aim was to evolve a method for predicting the resistance of a ship from resistance experiments made with a model of the vessel.

He tested geometrically similar models (geosims) in one of the first model towing tanks constructed at Torquay in the U.K..

He postulated that the total resistance of any hull, either ship or model, is the sum of two components; frictional resistance and residuary resistance:

$$R_T = R_F + R_R$$

He measured the resistance of thin planks, from 0.6m to 15.2m in length and since they were thin and had chamfered ends, he assumed that their resistance was due to friction only. He derived a formula for the frictional resistance of a plank of arbitrary length and surface area at a specified speed.

Froude's formula for the frictional resistance of a plank is:

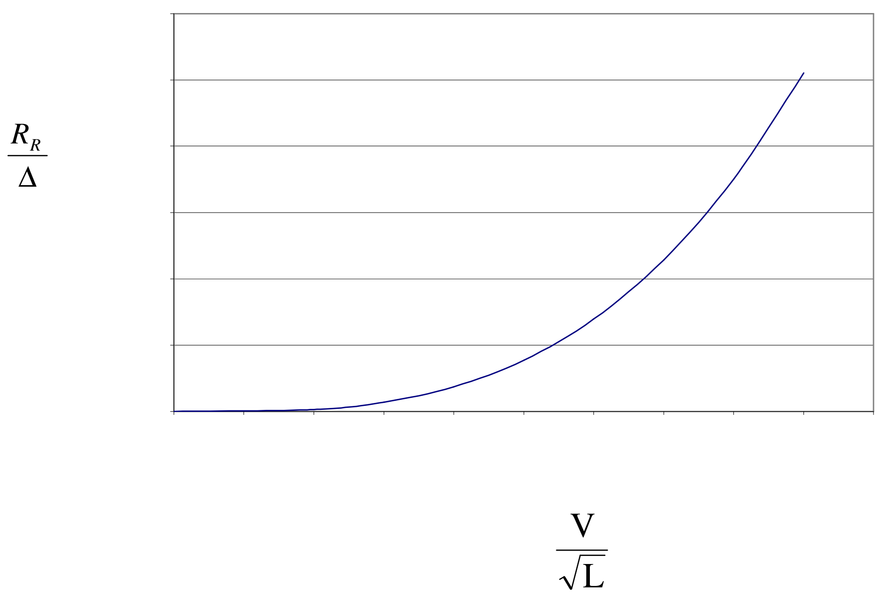
$$R_F = fSV^n$$

where f is a numerical constant, S is the wetted surface area and V the speed. The value for n was 1.825. This formula has been replaced by a modern equivalent.

Working on the assumption that the frictional component $R_F$ of the total resistance of his geosims equalled the resistance of a plank having the same length and area, he calculated the frictional resistance of each model at various speeds.

By subtracting these calculated values of $R_F$ from the appropriate measured values of $R_T$ he deduced the corresponding values of the residuary resistance $R_R$.

He then discovered that plotting values of residuary resistance per ton of displacement, $\frac{R_R}{\Delta}$ to a base of speed/length ratio, $\frac{V}{\sqrt{L}}$ with V in knots and L in feet, gave a unique curve for all geosims.



This led to the famous Law of Comparison which is neatly expressed in the form:

$$R_R \propto \Delta \quad \text{when} \quad V \propto \sqrt{L}$$

or

$$\frac{R_R}{\Delta} = f\left(\frac{V}{\sqrt{L}}\right)$$

Hence from the measured resistance of a model over a range of speeds he was able to predict the resistance of a geometrically similar model or ship.

## 5 Dimensional analysis of ship resistance

Dimensional analysis is a mathematical method which must be understood in order to work out how to scale ship resistance from model test results to full scale. It also provides insight to how components of resistance might interact.

### 5.1 Principles of dimensional analysis:

Dimensional analysis relies on the Buckingham-Pi theorem. Which states that:
*If you have n physical variables that describe a situation, and there are q principle dimensions, then the variables can be grouped into n-q independent dimensionless groups.*
The principal dimensions of all physical quantities are:

- Length L
- Mass M
- Time T

(quietly forgetting about light, sound etc. for the moment)

any equation which describes a physical occurrence must have the same physical dimensions regardless of the units used e.g.
the equation work done = force × distance moved
must be true for SI units, imperial units or any other unit system. By changing the equation around, we can reduce it to one containing only non-dimensional quantities:

*blank for you to complete*

### 5.2 Use of dimensional analysis (DA).

There are two ways that DA is often used. Firstly, you can check whether an equation might be correct by comparing the dimensions on the LHS with those on the RHS. If they are different, there is something wrong with the equation. (This is handy in exams when you are not sure if you have remembered an equation correctly. The second, and much more powerful use of DA is to solve physical problems. For example:

Q: What is the relationship between the resistance of a ship and it's dependent variables?

A: (the trick here is to identify all the independent variables without including irrelevant ones).

*Blank space for you to complete the answer*

## 6 The Scaling Problem:

Naked hull resistance essentially consists of three components:

- wavemaking
- skin friction
- viscous pressure

Wavemaking resistance is largely a function of Froude number whilst friction is Reynolds number dependent. Viscous pressure is Reynolds number dependent (as it is viscous in nature) but it is also affected by Froude number, and thus causes a few problems.
The ideal situation would be to run the model at the same Froude number and the same Reynolds number as the full size vessel, then the model resistance would be a simple, scale dependent ratio of the full size resistance. Unfortunately the laws of algebra do not allow this:

$$\text{Froude number} = \frac{v}{\sqrt{gL}}$$

$$\text{Reynolds number} = \frac{vL}{\nu}$$

where: v = vessel velocity (m/s)
L = waterline length (m)
$\nu$ = kinematic viscosity ($m^2/s$)

Assuming g and $\nu$ are the same for model and full size, then for constant Froude number:

$$v_m = v_s \sqrt{\frac{L_m}{L_s}}$$

whilst for constant Reynolds number:

$$v_m = v_s \frac{L_s}{L_m}$$

Therefore these two requirements are incompatible. A constant Reynolds number would result in ridiculously high speeds, so model tests are carried out at constant Froude number. This means that the model tests are carried out at very low Reynolds numbers.
This leads to two problems:
i) It is necessary to know how friction resistance varies with Reynolds number in order to scale the friction to full size. This problem has been addressed by the work of Froude and Hughes amongst others.
ii) If the Reynolds number is low the flow over the model may be laminar and not turbulent, this would have a large effect on the measured resistance.
To solve this scaling problem we need to know a bit more about the components of resistance.