Sailing catamaran performance metrics

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Nomenclature

A profile area (m^2)

A_b profile area of dagger board (m^2)

AR_e effective aspect ratio

A_s area of stub keel (m^2)

B_{OA} beam overall (m)

B_{wl} waterline beam of one hull (m)

C_L lift coefficient

$\frac{dC_L}{d_\alpha}$ lift curve slope (rad^{-1})

e non-dimensional lift

e_b non-dimensional lift of dagger board

e_s non-dimensional lift of stub keel

F_n Froude number

g acceleration due to gravity (m/s^2)

geosim geometrically similar shapes of different size

GZ righting arm (m)

h heeling lever (from VCB to VCE) (m)

k, k' arbitrary constants, sometimes dimensionless

L lift (sideforce) (N)

L_b dagger board lift (N)

L_s stub keel lift (N)

L_{OA} length overall (m)

L_{WL} length on waterline (m)

R_{low} resistance at low speed (Froude number) (N)

R_{high} resistance at high speed (Froude number) (N)

RM righting moment (Nm)

SA sail area (m^2)

T draft (m)

T_h hull draft excluding dagger boards (m)

V boat speed (m/s)

VCB vertical centre of buoyancy (m)

VCE vertical centre of effort of sails (m)

VCG vertical centre of gravity (m)

V_{ld} downwind light airs speed metric

V_{hd} downwind fresh breeze speed metric

V_{lu} upwind light airs speed metric

V_{hu} upwind fresh breeze speed metric

WSA wetted surface area (m^2)

α leeway angle (rad)

Δ mass displacement (kg)

ρ density of water (kg/m^3)

Pre-amble

In this article a set of five metrics are proposed for assessing the relative performance of cruising sailing catamarans, by using just six published design characteristics.

Cruising sailing catamarans are not the high-speed, easy-capsize racing foilers that capture the sailing public's attention. They are relatively heavy, sedate and stable live-aboard platforms. They are increasingly popular in the yacht charter market. In terms of design there are two main variants: those with retractable dagger boards and those with fixed stub keels. Their cruising role notwithstanding, performance is still an important attribute, and the usual optimisitic claims play an important part in their marketing. How can the average sailor cut through the sales spin and assess the relative performance of different models?

It is with some trepidation that I submit this technical note: whilst it attempts to provide something useful, it also flies in the face of good science or engineering. Almost since the beginnings of our profession, naval architects have tried to describe the complex shape of a vessel by reducing it to a few simple parameters – length, displacement, block coefficient and the like. We have also attempted the same with performance - resistance coefficient, Froude number, advance coefficient etc. Sometimes these efforts are underpinned by sound analytical processes such as dimensional analysis; at other times they are driven by pragmatism. The approach described here most definitely sits in the latter camp.

Assumptions

1. Rigs are geosims so $VCE = k \times \sqrt{SA}$
2. Overhang lengths are small and similar so $L_{WL} = k \times L_{OA}$
3. Hulls are approximately semi-circular section underwater, so $B_{wl} = 2 \times T_h$ This equation is not used directly, it merely supports the approximation that $WSA = k \times T_h \times L_{OA}$
4. At low Froude number, friction dominates drag, so $R_{low} = k \times WSA$
5. At high Froude number, wavemaking dominates drag so $R_{high} = k \times \Delta$
6. With wind forward of abeam, the sailing efficiency is governed by the underwater shape not the rig (most cruising cats have much lower hydrodynamic efficiency than aerodynamic efficiency) [ref 1].
7. We only have to consider one hull for lift, drag etc. provided it is done consistently.
8. In the absence of a published chord length for a dagger board, it is assumed to be half the board span.
9. Longitudinal stability is not taken into account. In practice this often sets an upper limit on downwind boat speed in a fresh breeze.

The equations

Fundamental relationship

The full velocity prediction process is simplified to:

$$speed = f\{stability, sail\ area, hull\ drag, foil\ efficiency\}$$

For comparison of boats of different sizes dimensionless numbers should be used for each of the above factors, with speed non-dimensionalised using Froude number:

$$F_n = \frac{V}{\sqrt{g \times L_{OA}}}$$

Strictly speaking waterline length should be used, however, for geosims it is acceptable to use the more-often published overall length.

The aim is to estimate comparative speed i.e. speed of one boat compared with speed of another, regardless of any size difference. Therefore once a performance factor that is dimensionless has been established, Froude's law can be used to obtain a metric for absolute speed.

Power to carry sail (tippiness factor)

We shall only concern ourselves here with small angle transverse stability. Small angle stability of a catamaran is easy to formulate because the centre of buoyancy shifts from the centreline to the outer hull as soon as the windward hull starts to lift. Furthermore, the VCG of catamarans has very little influence on small angle stability because the righting arm is so large. Also, the width of the hulls is small compared to the overall beam. Provided the analysis is limited to similar types of catamaran it can be assumed that the righting lever GZ is linearly proportional to the overall beam.

$GZ = k \times B_{OA}$ and

The righting moment $RM = k \times \Delta \times B_{OA}$

The heeling moment from the rig is the product of sail force and lever:

$$F_A = k \times (h \times SA)$$

As a first approximation $h = k\sqrt{SA}$

so the heeling moment $HM = k'SA^{1.5}$

The effort required to lift a hull – the “tippiness” – is linearly proportional to the ratio of heeling moment and righting moment i.e.

$$tippiness\ metric = \frac{100 \times SA^{1.5}}{\Delta \times B_{OA}}$$

The factor 100 is introduced to make the resulting metric easy to read and write. Note that this metric is not dimensionless.

Downwind speed in light winds

Two simplifying assumptions are made:

a) There is no leeway when sailing downwind so the efficiency of the foils plays no part in performance.

b) There is also no heeling moment, so stability plays no part either.

Drag in light airs is mostly from friction, and the thrust is proportional to sail area. Therefore boat speed is governed by the ratio of sail area to wetted surface area. The wetted surface area is linearly proportional to length, waterline beam and hull draft. Given the assumption of a circular cross section, a light airs downwind speed number can be written as:

$$V_{ld} = \frac{SA}{L_{OA}T_h} \times \sqrt{L_{OA}}$$

This assumes that a catamaran with boards will retract them when sailing downwind.

Downwind in fresh winds

As was the case in light airs, the same two simplifying assumptions can be made:

a) There is no leeway when sailing downwind so the efficiency of the foils plays no part in performance.

b) There is also no heeling moment so stability plays no part either.

Drag is mostly from wavemaking and the thrust is proportional to sail area. Froude's law states that wavemaking drag is linearly proportional to mass displacement for geosims. Therefore dimensionless boat speed is governed by a dimensionless ratio of sail area to displacement. Note that, because catamaran hulls are relatively slender, friction does make up a significant proportion of hull drag at high speeds and ought to be taken into account too. Perhaps that will be included in the next iteration of this work; simplicity is paramount for this first attempt.

$$V_{hd} = \frac{1000 \times SA^3}{\Delta^2} \times \sqrt{L_{OA}}$$

The factor 1000 is introduced to make the resulting number easy to read and write.

Upwind hull efficiency

Now that the basic drag and stability characteristics have been identified, the remaining and most complex task is to determine the other factors affecting windward performance. This can be reduced to estimating the lift-drag ratio of the underwater hull shape. There are two main types of catamaran underwater hull shape – those with retractable dagger boards and those with fixed stub keels. It is assumed that the hull drag is the same for both configurations. Therefore the difference in efficiency is attributed only to their ability to generate lift (sideforce).

The basic lift equation is:

$$L = C_L \frac{1}{2} \rho A V^2$$

This immediately creates a problem – the solution is iterative, requiring an estimate of boat speed V before we can calculate the lift, which determines boat speed. As a first approximation it is assumed that boat speed is the same for all boats. On that basis, the two determining factors for producing lift are lifting area A and lift coefficient C_L. It is at this point that each underwater configuration must be examined separately.

Hull with stub keels

From slender body theory [ref 2], for typical very low aspect ratio stub keels:

$$\frac{dC_L}{d_\alpha} = \frac{\pi}{2}$$

Therefore:

$$L_s = k' A_s \frac{\pi}{2}$$

Estimating the area of the stub keel from published data might at first seem problematic. However, for slender bodies such as catamaran hulls the hull itself contributes a useful amount of lift, as well as the stub keel. Therefore the entire underwater shape can be treated as one big slender body (or, if you prefer, one big stub keel). Provided hulls with similarly proportioned stub keels are being compared, the lifting area can be considered directly proportional to both the hull length and the total draft (including the stub keel). Therefore:

$$L_s = k' L_{OA} \times T \times \frac{\pi}{2}$$

If we assume the stub keel is half the length of the boat and half the draft, then:

$$L_s = k' \frac{L_{OA}}{2} \times \frac{T}{2} \times \frac{\pi}{2}$$

This is not a dimensionless quantity. In order to non-dimensionalise it must be divided either by displacement or length cubed (we can ignore the g and ρ). The amount of lift generated has arguably less to do with mass than length, so length is chosen:

$$e_s = \frac{L_s}{{L_{OA}}^3} = k' \frac{1}{{L_{OA}}^2} \times \frac{T}{8} \times \pi$$

It is again assumed that the induced drag from the stub keels is small relative to the other drag components of the hull. That is not a very good assumption; it needs to be included in the next iteration of this work.

Hull with retractable boards

For aspect ratios typical of dagger boards, low aspect foil theory and empirical data [ref 3] show that:

$$\frac{dC_L}{d_\alpha} = \frac{2\pi}{\left(1 + \frac{3}{AR_e}\right)}$$

For most board configurations at moderate boat speeds the effective aspect ratio is consistently about twice the geometric aspect ratio, so for this type of analysis geometric aspect ratio can be used. Furthermore, to a very crude first approximation for typical board aspect ratios, the lift curve slope is directly proportional to aspect ratio (try for yourself by calculating it for effective aspect ratios of 1.5 and 3).

For a typical board of geometric aspect ratio 1.5 (effective aspect ratio about 3):

$$\frac{dC_L}{d_\alpha} = \pi$$

so the lift equation for the board becomes:

$$L_b = k' A_b \pi$$

It is assumed that the induced drag from the board is small relative to all the other drag components of the hull. This is probably a reasonable assumption, given the high efficiency (hence high lift-drag ratio) of a board.

The hull of a boat with retractable boards also contributes to lift, just as it does for a boat with stub keels. The total lift is therefore:

$$L_{tot=} k' A_b \pi + k' \frac{L_{OA}}{2} \times \frac{T_h}{2} \times \frac{\pi}{2}$$

and the non-dimensional lift is:

$$e_b = \frac{L_b}{{L_{OA}}^3} = \frac{\left(k' A_b \pi + k' L_{OA} \times T_h \times \frac{\pi}{8}\right)}{{L_{OA}}^3}$$

Readers who are still awake at this point may realise I have committed the unforgivable sin of adding two quantities that are dimensionally consistent but arithmetically unrelated - the constants of proportionality contain different parameters. The two weak defences offered for doing this are:

a) it seems to yield believable results; and

b) I have not yet found a better way of dealing with it.

Upwind in light winds

In light winds, power to carry sail is not relevant, and drag is mostly friction drag.

Therefore the important parameters for upwind sailing are sail area, wetted surface area and foil lift:

$$V_{lu} = k \times \left(\frac{SA}{WSA}\right)^{a} \times (e)^{b} \times \sqrt{L_{OA}}$$

(use e_s for the stub keel and e_b for the retractable board.)

Upwind in fresh winds

In a fresh breeze two things change:

a) the power to carry sail becomes important, and

b) drag is mainly from wavemaking, not much from friction.

Therefore:

$$V_{hu} = 10 \times k \times \left(\frac{SA^3}{\Delta^2}\right)^{a} \times (e)^{b} \times \left(\frac{\Delta \times B_{OA}}{SA^2}\right)^{c} \times \sqrt{L_{OA}}$$

The factor 10 is introduced to make the resulting number easy to read and write.

The upwind equation power indices

The values of the indices a, b and c in both of the two upwind metrics are not known. It is quite possible that the indices a and b are different in each equation; however, for this first attempt it is assumed they are the same.

Their values were determined empirically by comparing the output numbers for an idealised test boat with and without boards, and in lightship and full load. All three indices were initially set to unity but this resulted in performance differences that were unrealistic. Trial and error was then used to obtain plausible results on the test boat. This was achieved by changing index a to 0.5, with indices b and c remaining at 1.0.

Error sources

1. The biggest uncertainty is probably the estimation of displacement. Most published figures do not state whether they are lightship or full load; the difference is typically 30%. If both load conditions are known, they can be treated as separate boats.

2. The second biggest uncertainty is the estimation of sail area. Whilst it would be reasonable to assume the published data is for upwind sail area, some data uses the area of a non-overlapping jib whereas other data appears to use an overlapping genoa. The difference is typically 15-20% of total sail area.
3. The importance of foil efficiency and transverse stability decrease as apparent wind angle (AWA) increases because leeway decreases with AWA increase. The decrease is dealt with as a step function – one metric for upwind sailing, another for downwind sailing. Clearly this is a poor way of dealing with beam-reaching performance, but a weighted average of the two might offer a useful indication.

Results

The performance metrics for a dozen production catamarans have been calculated and the results seem plausible. However, I do not have enough confidence in them to publish the results just yet. Nevertheless, to give some indication of what might be, here are the results for three idealised boats.

Design	**test 1 lightship**	**test 1 full load**	**test 1 light + boards**
Δ (kg)	3000	4000	3000
L_{OA} (m)	10	10	10
B_{OA} (m)	5	5	5
T (m)	1	1	0.7
SA (m^2)	50	50	50
board span (m)	-	-	1.5
board chord (m)	-	-	0.75
tippiness metric*	2.4	1.8	2.4
V_{ld} **	16	16	23
V_{lu} **	9	9	17
V_{hd} **	44	25	44
V_{hu} **	28	28	44

* A high value of tippiness metric means the boat is tippy.

** A high value of speed metric means the boat is fast.

Conclusions

It would be foolish to offer conclusions from such tentative work as this. The reason for publishing this technical note is to canvass views as to whether, despite the numerous assumptions and approximations, the approach taken has merit; or is it too far removed from reliable naval architecture? I look forward to your responses!

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